ASSIGNMENT I

Problem 1. Given an undirected graph $G = (V, E)$, the *minimum vertex cover problem* asks for a minimum set $S \subseteq V$ such that for every edge $e \in E$, $e \cap S \neq \emptyset$. If we use $x_v \in \{0, 1\}$ to indicate whether v is in S, the following integer program is equivalent to computing minimum vertex cover:

$$\begin{aligned} \text{minimize} \quad & \sum_{v \in V} x_v \\ \text{s.t.} \quad & x_u + x_v \geq 1, \quad \forall \{u, v\} \in E \\ & x_v \in \{0, 1\}, \quad \forall v \in V \end{aligned}$$

Please relax the above integer program to a linear program and apply linear programming rounding approach to obtain a 2-approximation algorithm for minimum vertex cover problem.

Problem 2. We have shown in class that the minimum label s-t cut problem has an $O\left(\sqrt{\frac{|E|}{\mathbf{OPT}}}\right)$-approximation algorithm. The algorithm is obtained in the following way:

(1) round the LP solution to get a partial cut S;
(2) remove S and compute the minimum s-t cut in the remaining graph.

In the class, we used Menger's theorem to bound the size of minimum s-t cut in step (2). In fact, we can bound the size of the minimum cut in another way: use BFS to layerize vertices from s to t, and bound the number of edges between layers. Please use this idea to obtain an approximation algorithm with approximation ratio only depending on $|V|$ and **OPT**.

Problem 3. Determine the integrality gap of our MaxCut vector programming relaxation. Prove the best bound you can find.